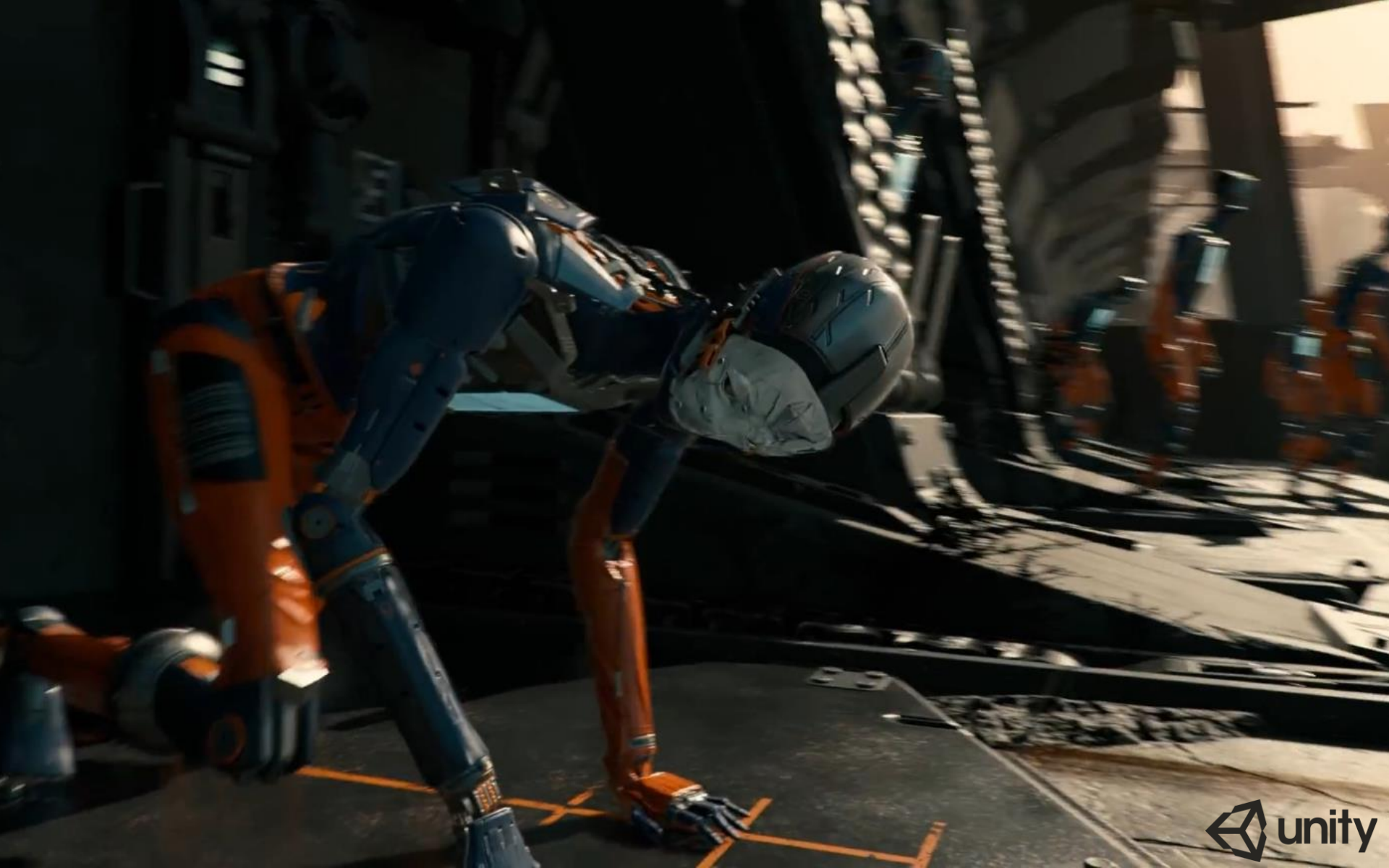


# Physically-Based Rendering

Shih-Chin Weng

[shihchin.weng@gmail.com](mailto:shihchin.weng@gmail.com)

What is PBR?







**SOLIDANGLE**

The Chemical Brothers - Wide Open, The Mill

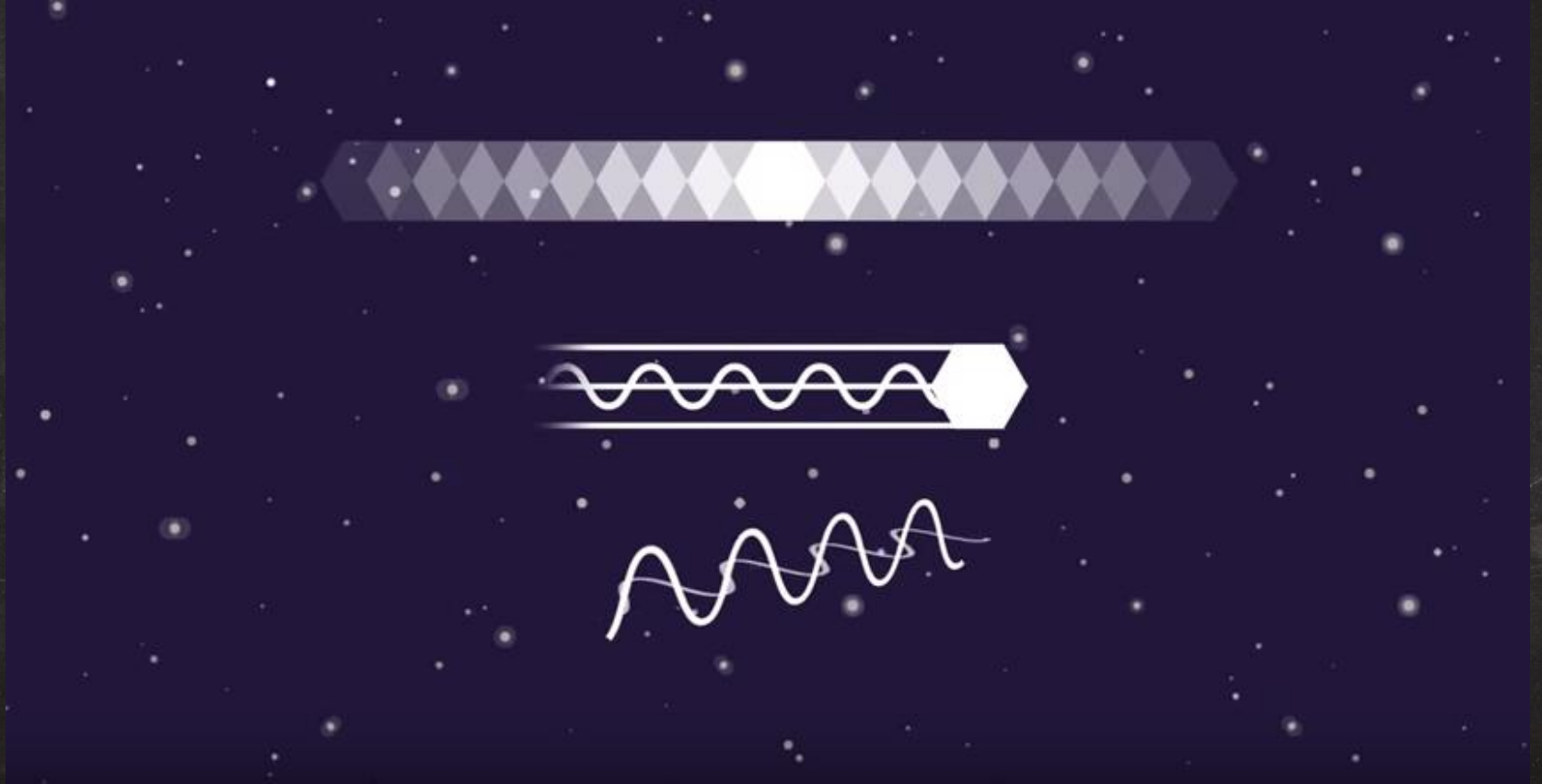
# *Physically **B**ased **R**endering*

*Simulate materials and lights based on physical laws or observations of real world more accurately.*

# Stages of Photorealistic Rendering

1. Measurement and acquisition of scene data
  - BRDF, BSSRDF, BTF, etc.
2. Light transport simulation
  - Ray tracing, photon-mapping, radiosity, etc.
3. Visual display
  - Tone mapping

# What Is Light?





# ELECTROMAGNETIC SPECTRUM

GAMMA RAYS

X-RAYS

ULTRAVIOLET

INFRARED

RADAR

FM

TV

AM

INVISIBLE TO THE HUMAN EYE

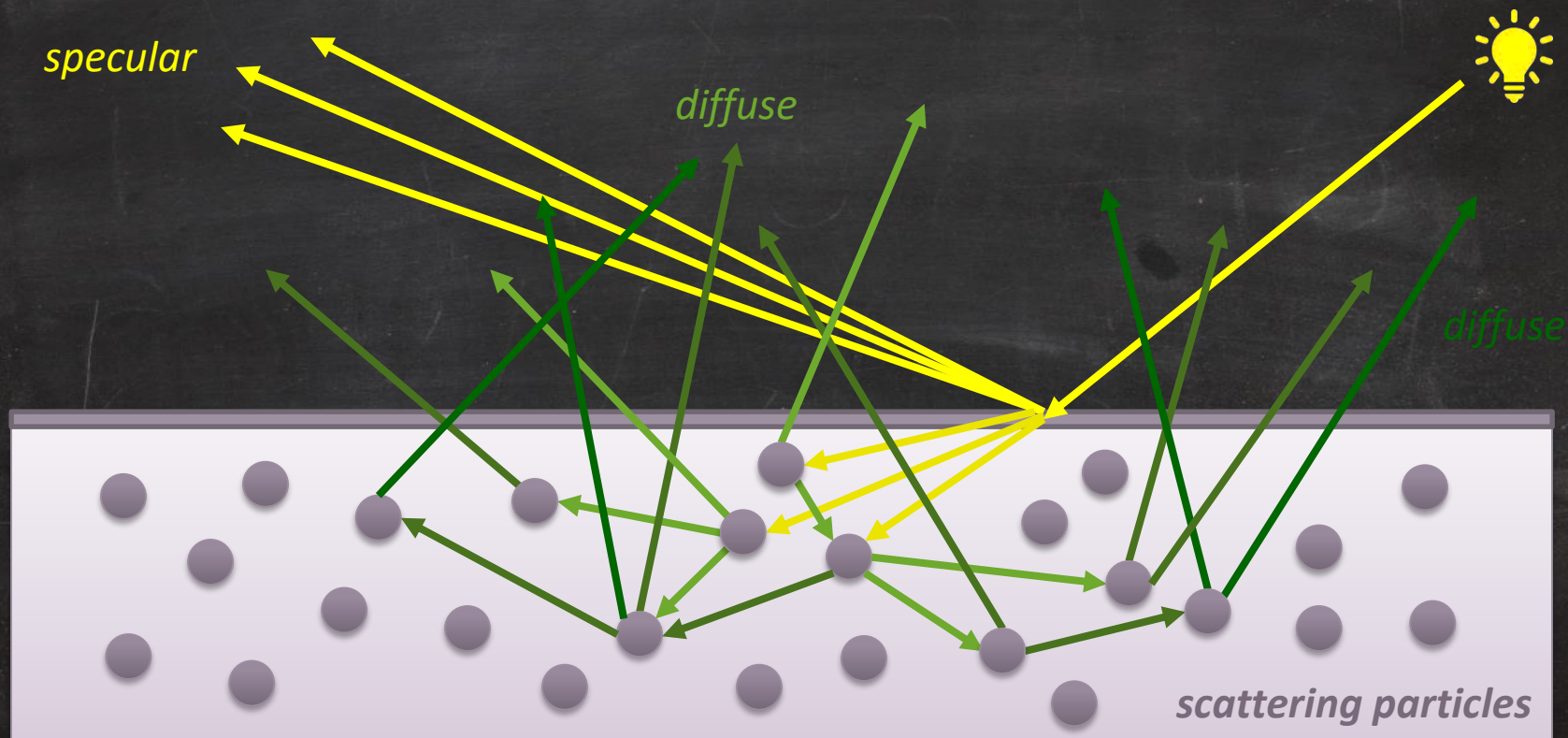
INVISIBLE TO THE HUMAN EYE



# Geometric Optics

- Assumption: the wavelength of light is much smaller than the scale of interacted object
- Light travels
  - in **straight** lines
  - **instantaneously** through a medium
- Light is **not** influenced by gravity or magnetic fields
  - No diffraction, dispersion
  - But the movie “Interstellar” does simulate the light bent by gravity!!

# Light Matter Interaction



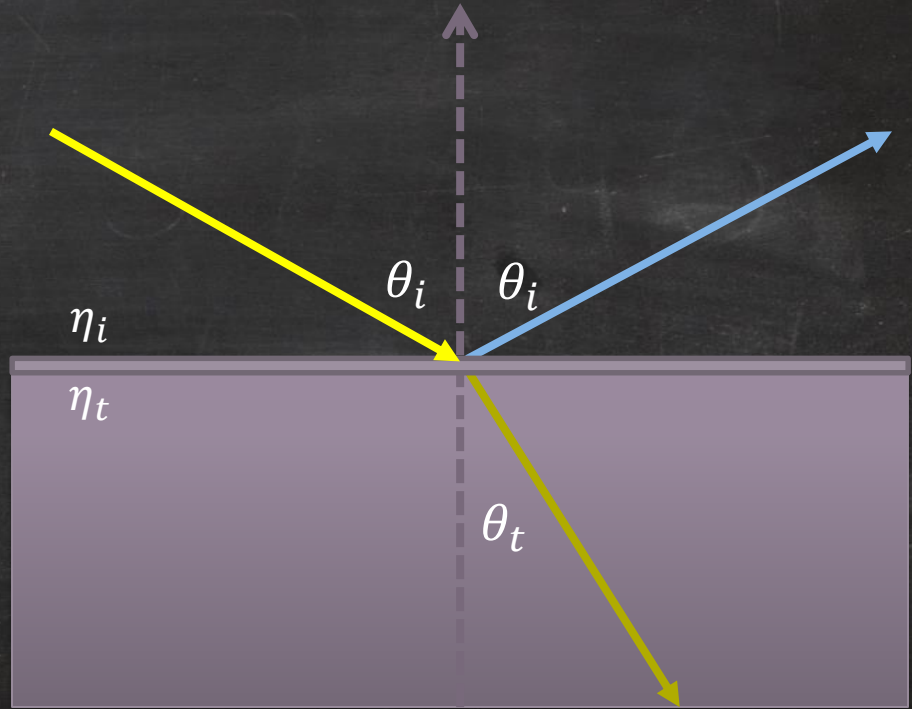
Snell's Law



# Snell's Law

$$\sin \theta_i \eta_i = \sin \theta_t \eta_t$$

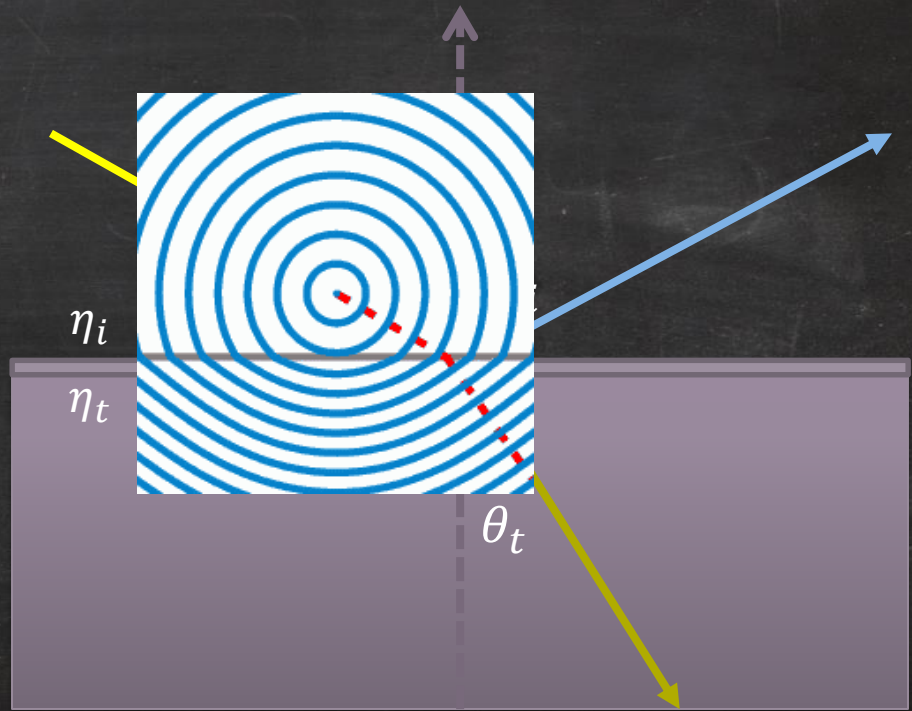
Index of Refraction (IOR):  $\eta$



# Snell's Law

$$\sin \theta_i \eta_i = \sin \theta_t \eta_t$$

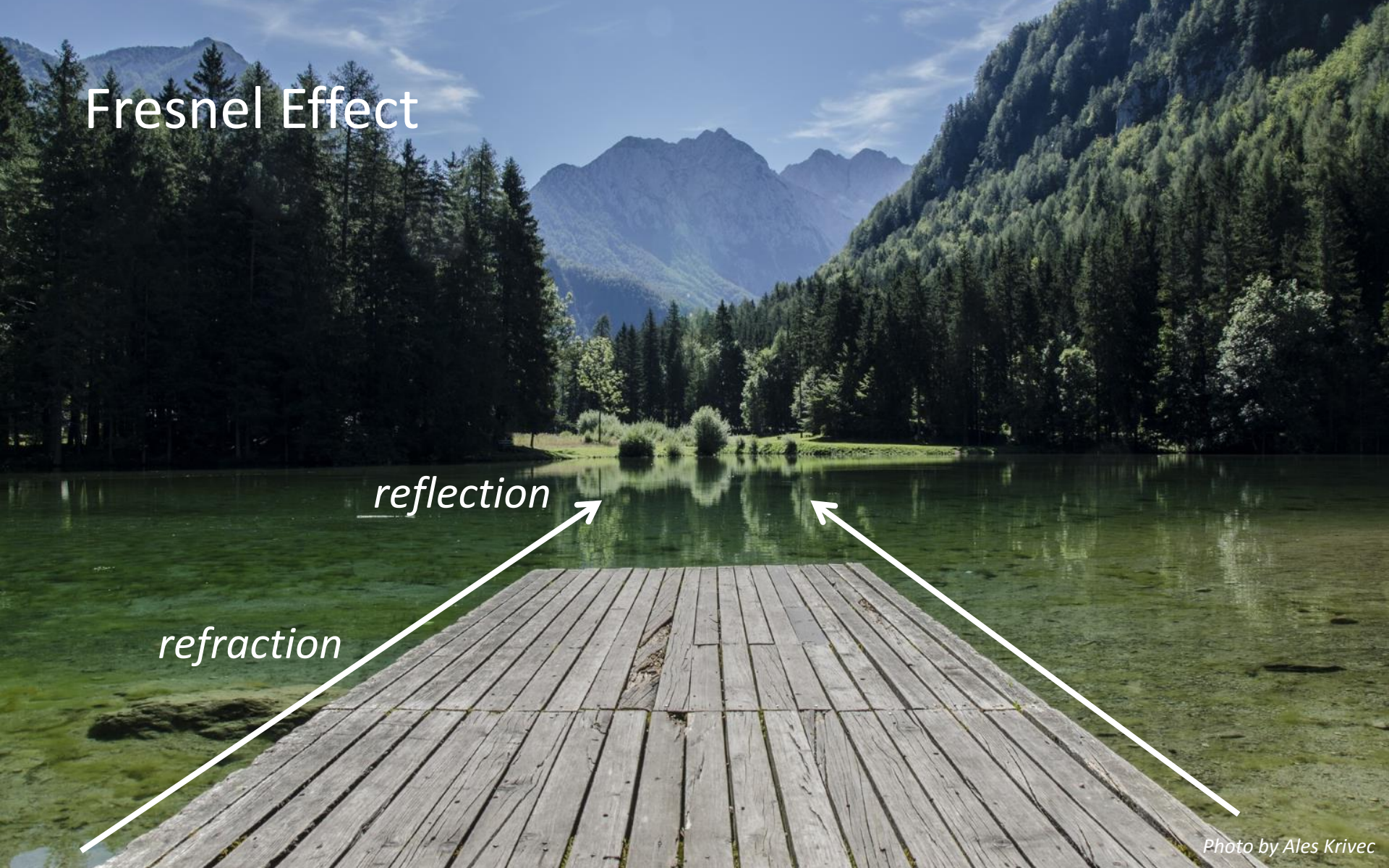
Index of Refraction (IOR):  $\eta$



# Fresnel Effect

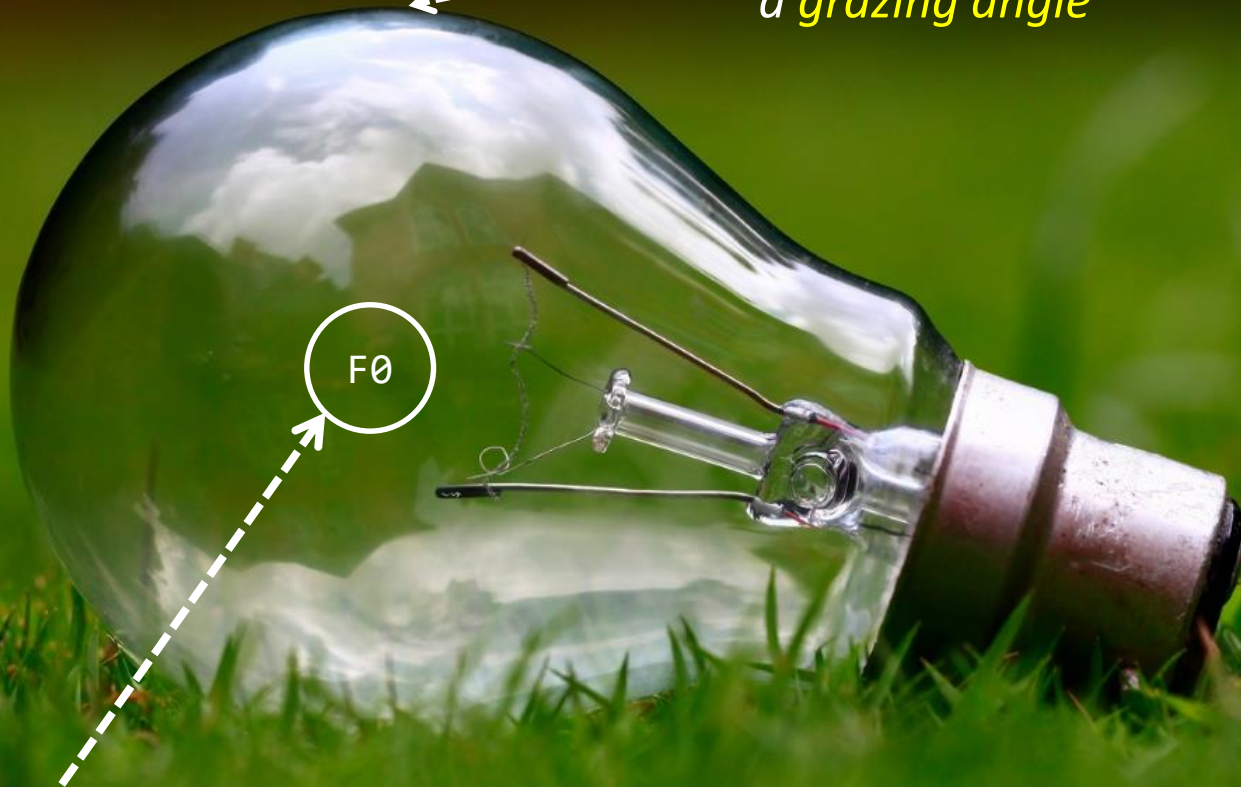
reflection

refraction



# Fresnel Effect

*more and more reflective  
as the angle of view approaches  
a **grazing angle***



*reflectance at normal*



# Fresnel

- Fresnel reflectance
  - the amount of reflected light w.r.t. the viewing angle
- Relates the ratio of **reflected** and **transmitted** energy as a function of
  - Incident direction
  - Polarization
  - Materials' properties

# Material Properties

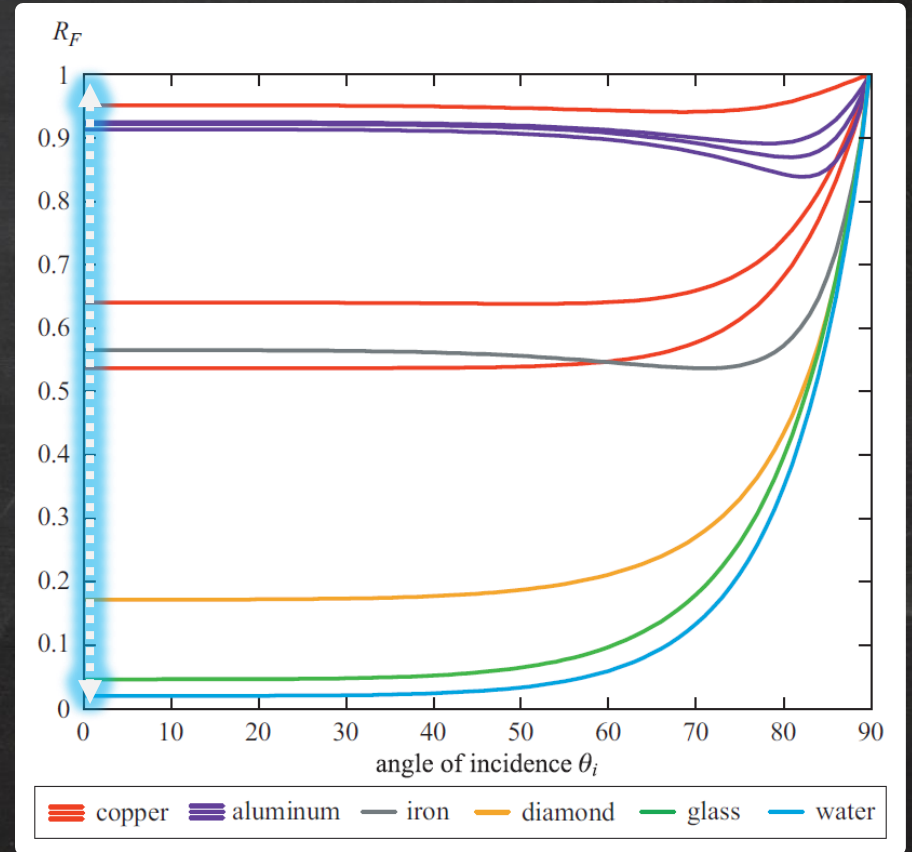
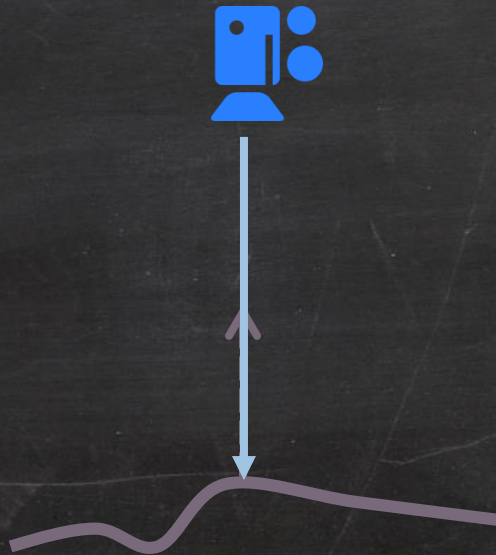
## Non-metal (dielectrics)

- Only reflect 4~10% of incoming light in average
- The reflection intensity is independent on the wavelength
- No energy is absorbed during reflection
  - but might be absorbed during subsurface scattering

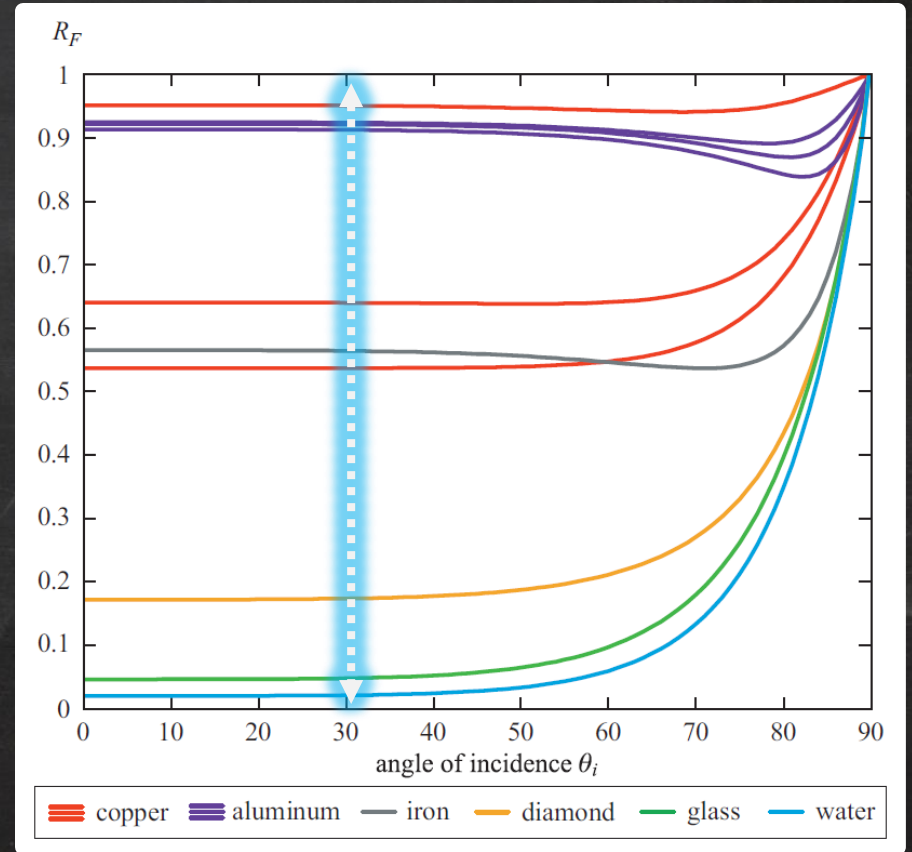
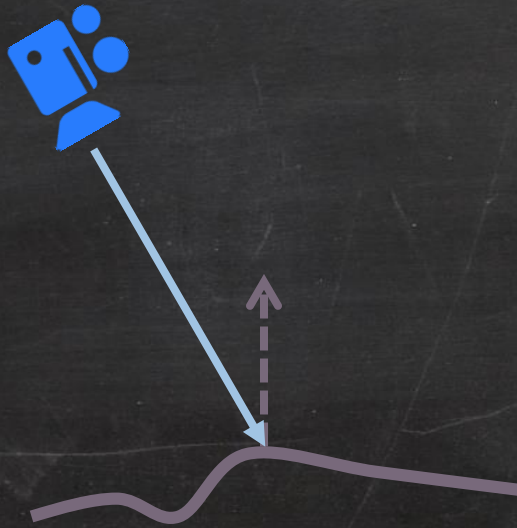
## Metal

- IOR strongly depends on the wavelength
- **Immediately absorbs** refracted lights (i.e. no refraction)
  - The reflected lights would change their color

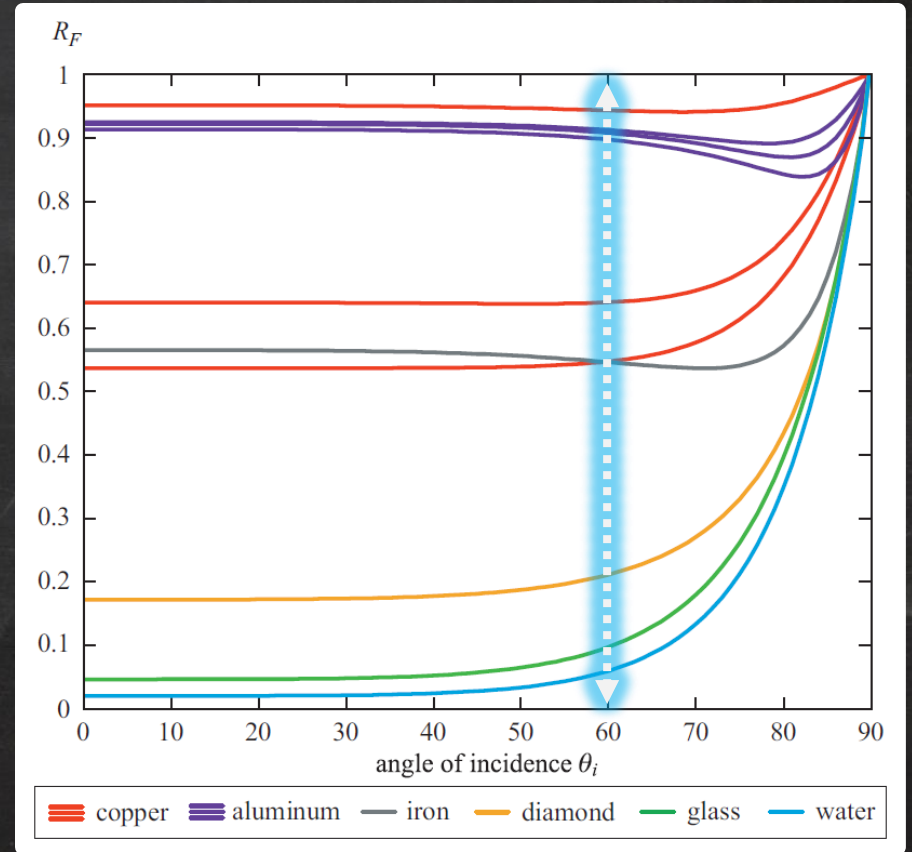
# Fresnel Reflectance



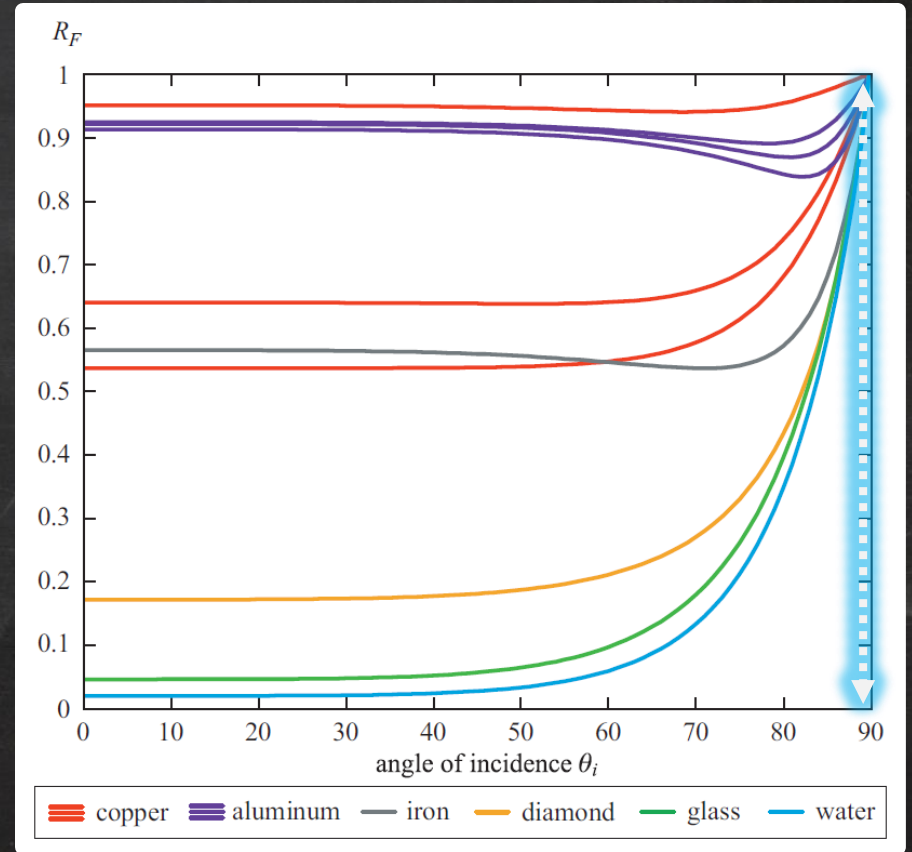
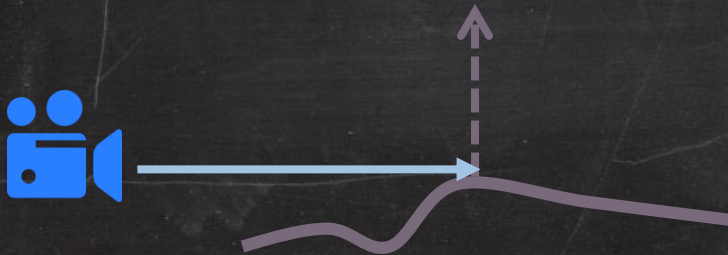
# Fresnel Reflectance



# Fresnel Reflectance

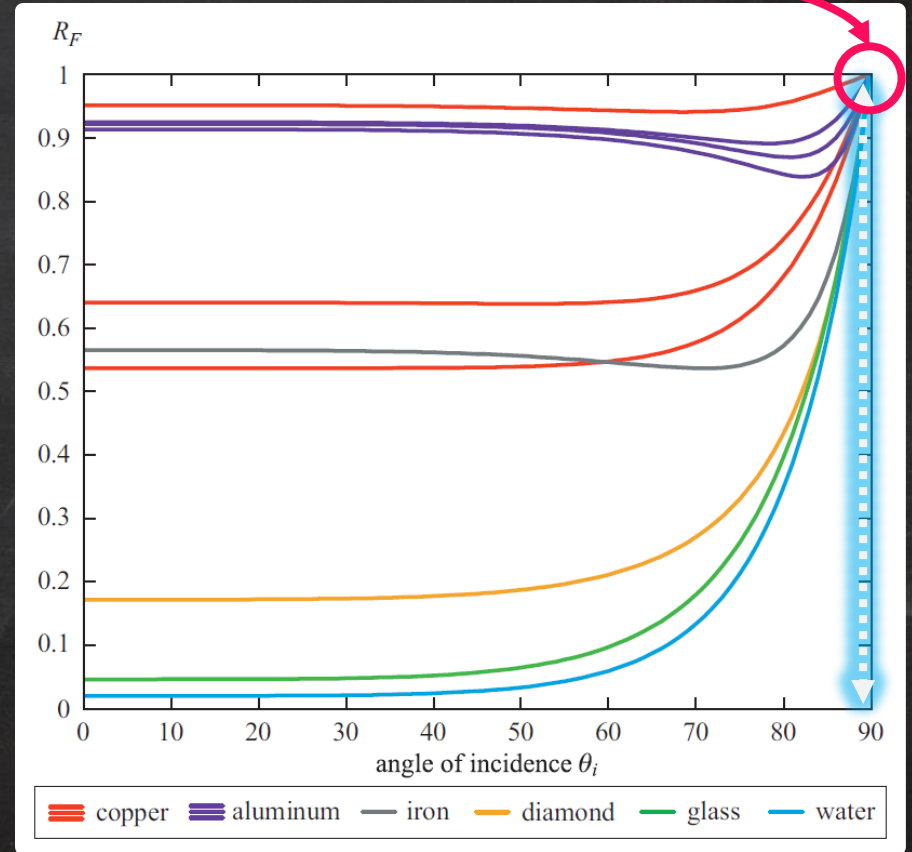
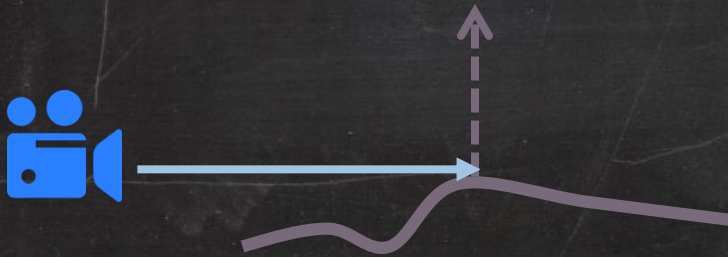


# Fresnel Reflectance



# Fresnel Reflectance

Reflection goes to **100%** at grazing angle!



# Fresnel

for unpolarized light

$$F_r = \frac{1}{2} (r_{\parallel}^2 + r_{\perp}^2)$$

Dielectric

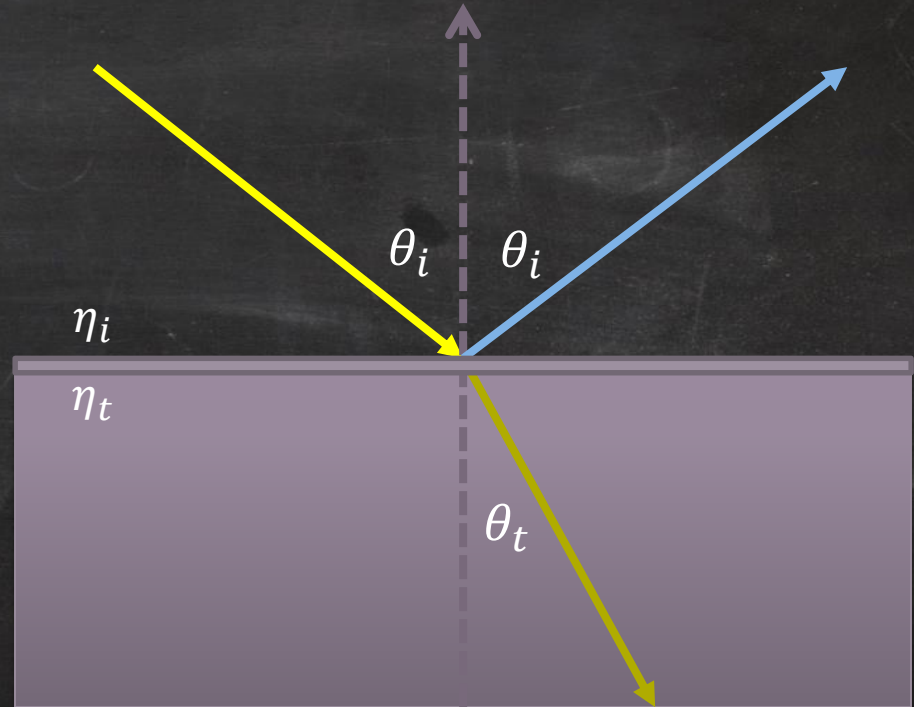
$$r_{\parallel} = \frac{\eta_t \cos \theta_i - \eta_i \cos \theta_t}{\eta_t \cos \theta_i + \eta_i \cos \theta_t}$$

$$r_{\perp} = \frac{\eta_i \cos \theta_i - \eta_t \cos \theta_t}{\eta_i \cos \theta_i + \eta_t \cos \theta_t}$$

Conductor

$$r_{\parallel}^2 = \frac{(\eta^2 + k^2) \cos^2 \theta_i - 2\eta \cos \theta_i + 1}{(\eta^2 + k^2) \cos^2 \theta_i + 2\eta \cos \theta_i + 1}$$

$$r_{\perp}^2 = \frac{(\eta^2 + k^2) - 2\eta \cos \theta_i + \cos^2 \theta_i}{(\eta^2 + k^2) + 2\eta \cos \theta_i + \cos^2 \theta_i}$$





# Radiometry

$$\text{Radiant flux } \Phi = \frac{dQ}{dt} \text{ (J/sec)}$$

The total amount of energy passing through **a region of surface** per **unit time**

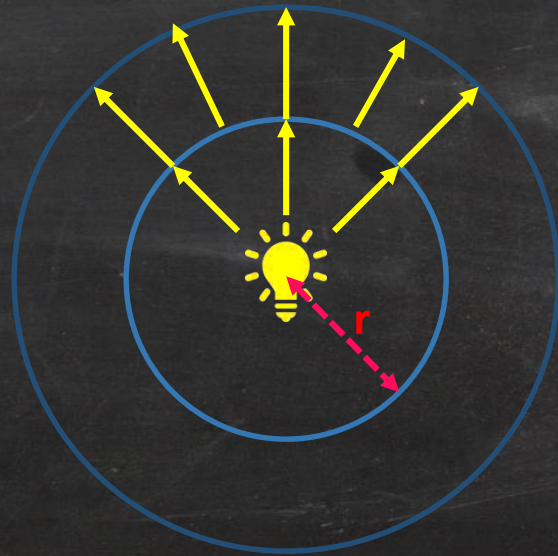
$$\text{Irradiance } E = \frac{d\Phi}{dA}$$

Pre area **incoming** flux at a surface

Radiant Exitance or Radiosity

$$M = B = \frac{d\Phi}{dA}$$

the total amount  $\Phi$  measured at **inner** and **outer** sphere is **the same** (equals to the radiant flux of the point light)



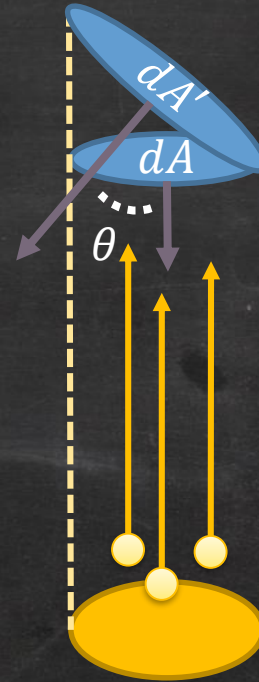
$$E = \frac{\Phi}{4\pi r^2}$$

# Lambert's Cosine Law

$$E = \frac{d\Phi}{dA}$$



$$E_1 = \frac{d\Phi}{dA}$$



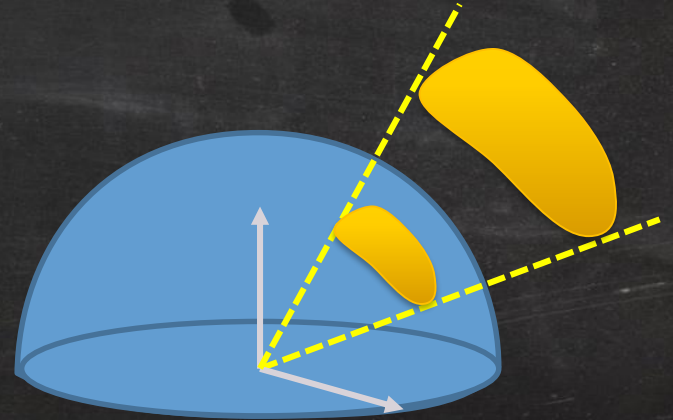
$$dA = dA' \cos \theta$$

$$E_2 = \frac{d\Phi}{dA'} = \frac{\cos \theta d\Phi}{dA} = E_1 \cos \theta$$

# Solidangle

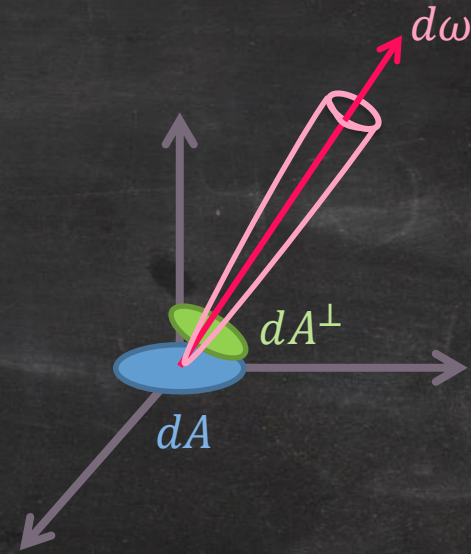
$$\Omega = \frac{A}{r^2}$$

- The total area on a **unit sphere** subtended by the object
- A set of *directions*
- Measured in steradians (sr)
- Often denoted as  $\omega$



# ★ Radiance

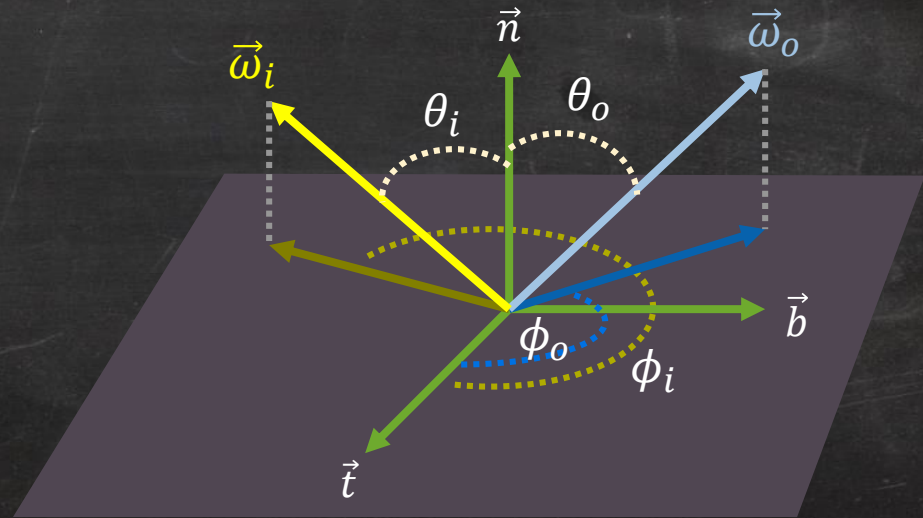
$$L = \frac{\overset{\text{flux}}{d^2\Phi}}{\underset{\text{solid angle}}{d\omega} \underset{\text{projected area}}{dA^\perp}} = \frac{d^2\Phi}{d\omega dA \cos\theta}$$



The density of photons passing *near x* and traveling in directions near  $\omega$

# Bidirectional Reflection Distribution Function

$$f(\theta_i, \phi_i, \theta_o, \phi_o) = f(\vec{\omega}_i, \vec{\omega}_o)$$



# BRDF Definition

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{dL_r(\vec{\omega}_o)}{dE_i(\vec{\omega}_i)} = \frac{dL_r(\vec{\omega}_o)}{L_i(\vec{\omega}_i) \cos \theta_i d\omega_i}$$

Diagram illustrating the BRDF definition:

- The numerator  $dL_r(\vec{\omega}_o)$  is labeled "outgoing radiance" (blue arrow).
- The denominator  $dE_i(\vec{\omega}_i)$  is labeled "incoming irradiance" (pink arrow).

# BRDF Definition

spending  
income

outgoing radiance

$$f(\vec{\omega}_i, \vec{\omega}_o) = \frac{dL_r(\vec{\omega}_o)}{dE_i(\vec{\omega}_i)} = \frac{dL_r(\vec{\omega}_o)}{L_i(\vec{\omega}_i) \cos \theta_i d\omega_i}$$

incoming irradiance

# Properties of BRDFs

- Helmholtz reciprocity

- symmetric surface reflectance

$$f(\vec{\omega}_i, \vec{\omega}_o) = f(\vec{\omega}_o, \vec{\omega}_i)$$

- Positivity

$$f(\vec{\omega}_i, \vec{\omega}_o) \geq 0$$

- Energy conservation

- Total amount of outgoing energy must be *less than or equal to* the incoming energy



# BRDF Explorer

from Disney Animation

<http://www.disneyanimation.com/technology/brdf.html>

The screenshot displays the BRDF Explorer software interface, which is used for configuring and visualizing Bidirectional Reflectance Distribution Functions (BRDFs). The interface is divided into several panels:

- Theta H Plot:** A graph showing the distribution of light reflection across different angles. The x-axis represents the angle from 0 to 90 degrees, and the y-axis represents the reflectance from 0.05 to 0.4. A blue curve and a yellow stepped curve are visible.
- Phi V:** A control for the phase angle, currently set to 51.12 degrees.
- 3D Plot:** A 3D visualization of a yellow sphere being lit by a blue light source. Red and cyan lines indicate the incident and reflected rays.
- Lit Object:** A 3D rendering of a white teapot in a desert-like environment, demonstrating the BRDF's effect on a real-world object.
- BRDF Parameters:** A panel for adjusting various parameters:
  - Luminance:** A dropdown menu.
  - Log plot:**  $y = \log_{10}(x + 1.0)$
  - Multiply by N · L:** A checkbox.
  - Incident angle - thetaL:** A slider set to 45.
  - Incident angle - phiL:** A slider set to 45.
  - Material Selection:** A list with "white-marble.binary" and "ashikhman\_shirley.brdf".
  - Rs:** A checkbox and a slider set to 0.023.
  - Rd:** A slider set to 0.195.
  - nu:** A slider set to 100.
- Lit Sphere:** A 2D visualization of a sphere showing the lighting effect, with checkboxes for "Double theta" and "Multiply by N · L".
- Gamma:** A slider set to 2.2.
- Exposure:** A slider set to 0.
- Polar Plot:** A graph showing the BRDF's response in polar coordinates, with multiple colored curves.

At the bottom, there are tabs for "Image Slice" and "Lit Object", and a "Keep Sampling" checkbox.

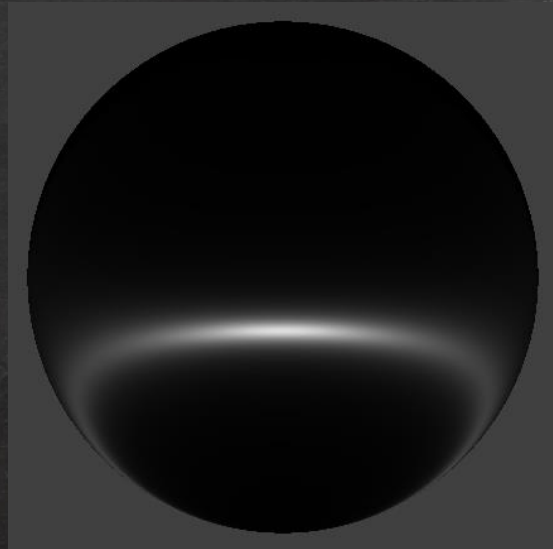
[Image courtesy of Disney.]

# Isotropic vs. Anisotropic

- Isotropic BRDFs are independent of incident azimuth angle  $\phi$

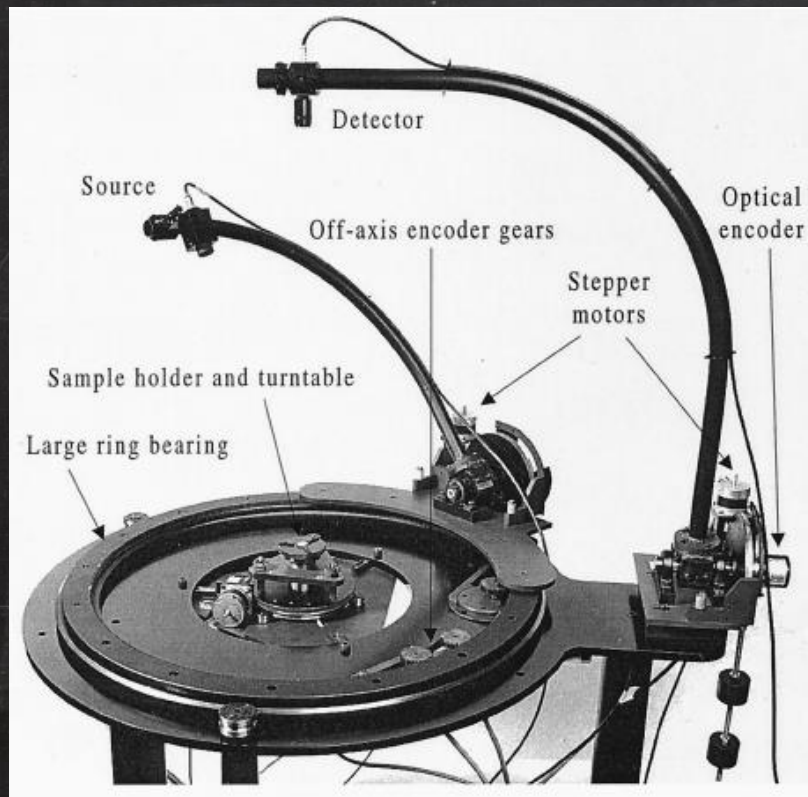


isotropic

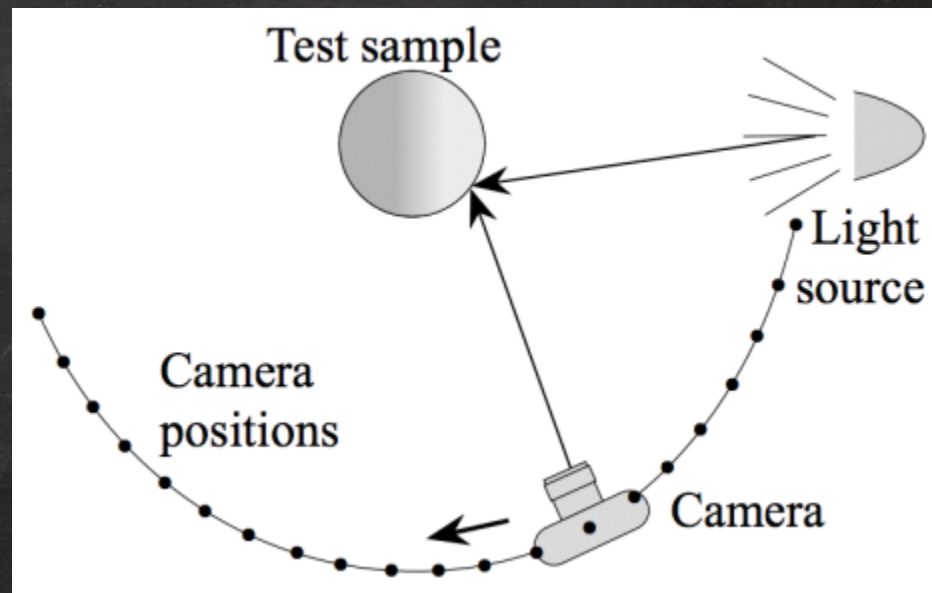


anisotropic

# BRDF Acquisition



[White et al, JAO 98]

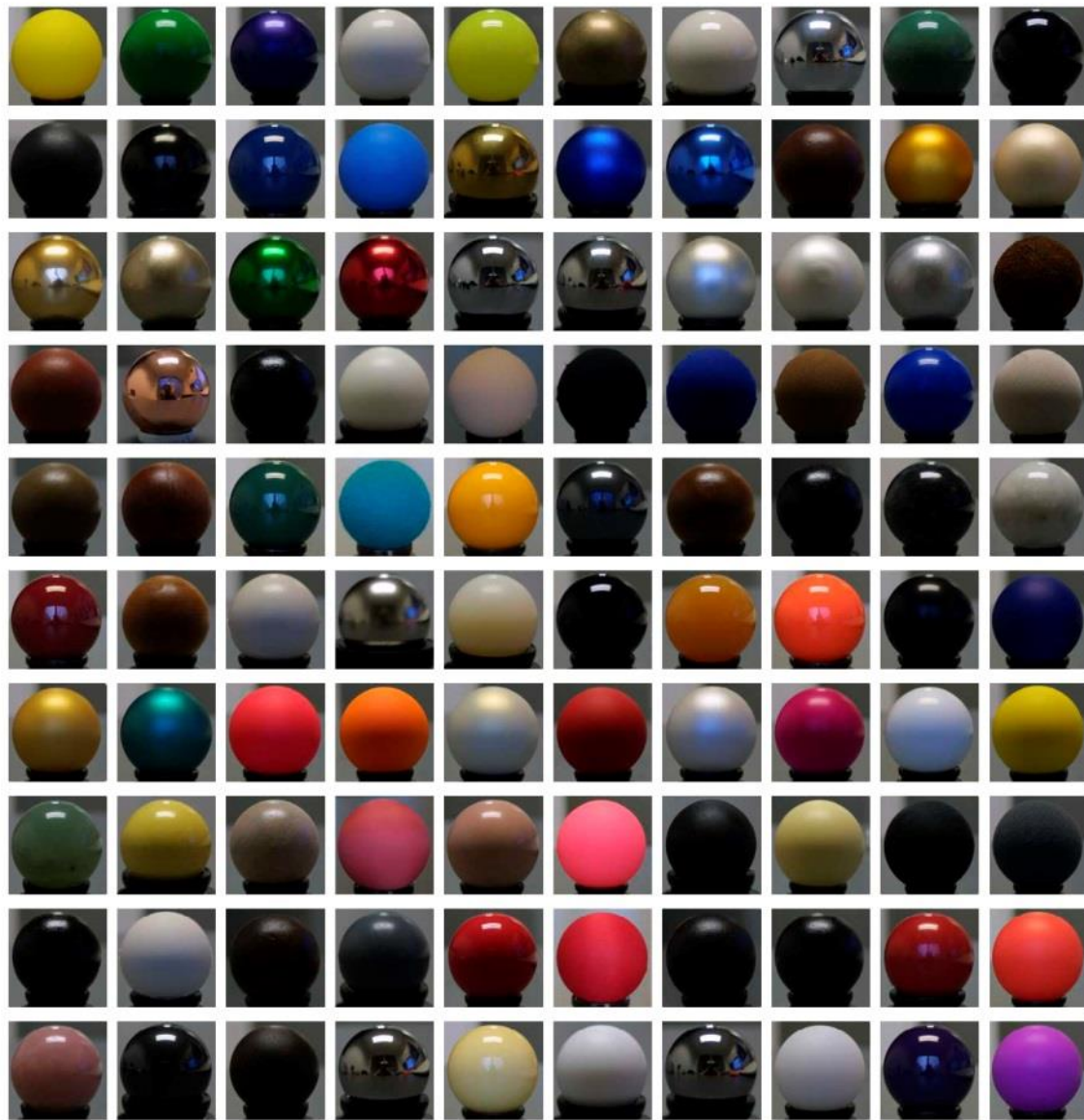


[Marschner et al. 1999]

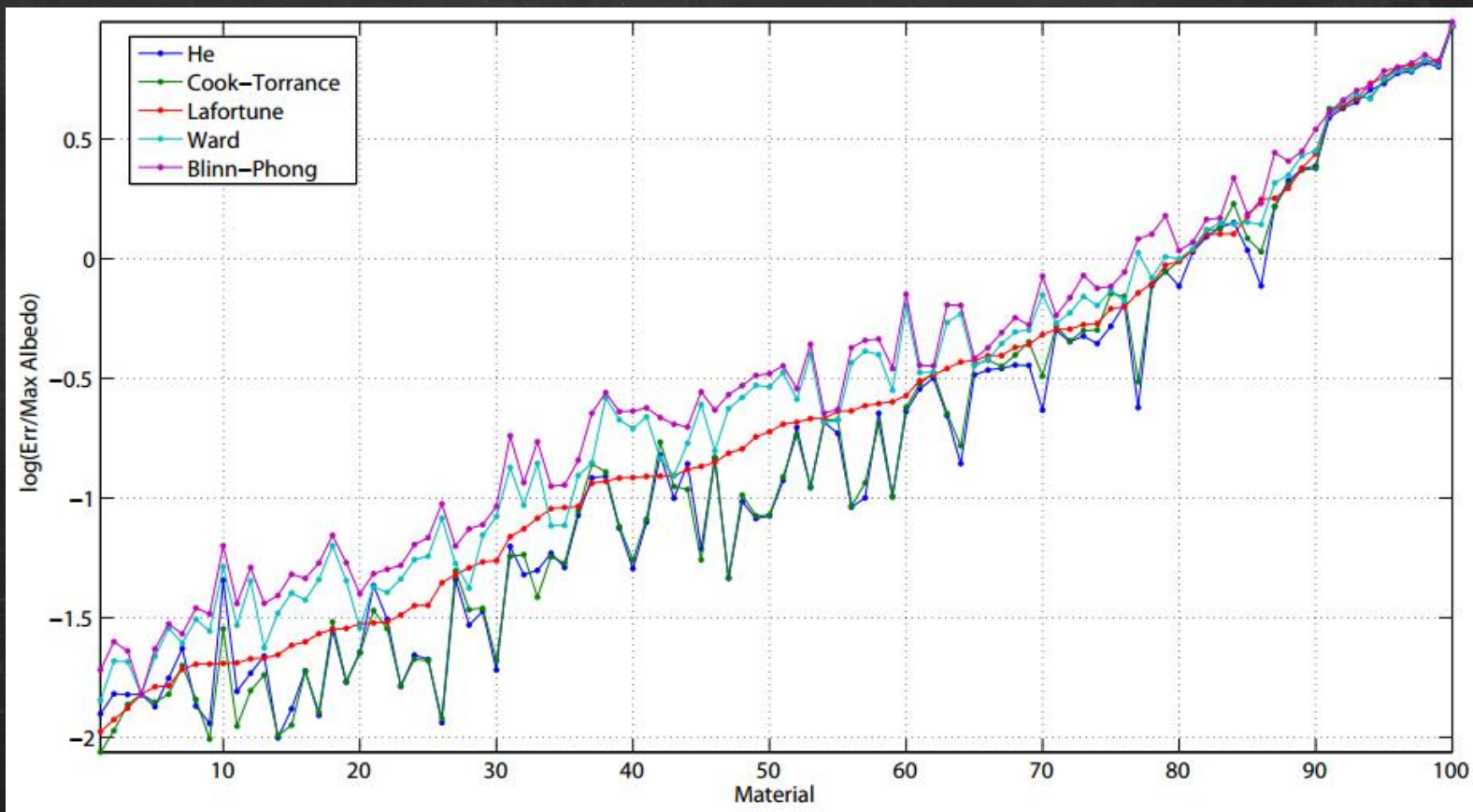
# MERL 100

<http://www.merl.com/brdf/>

*“A Data-Driven Reflectance Model”*,  
Matusik et al., SIG'03

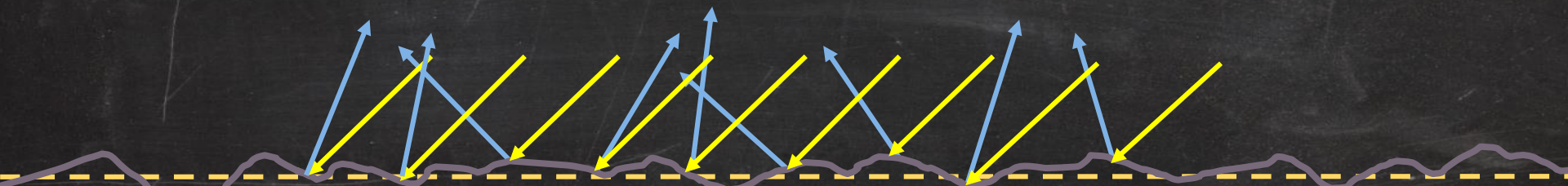


# BRDF Data Fitting



# Microfacet Model

# Microfacet Model



macrogeometry

$$\vec{h} = \frac{\vec{l} + \vec{v}}{\|\vec{l} + \vec{v}\|}$$

# Microfacet Model



macr  
geometry

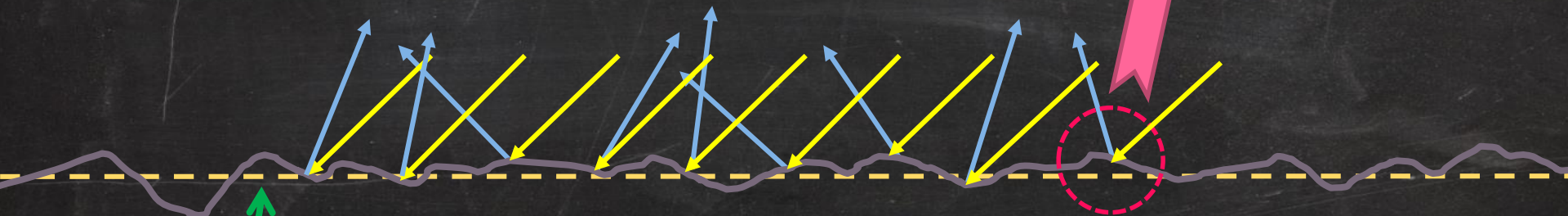
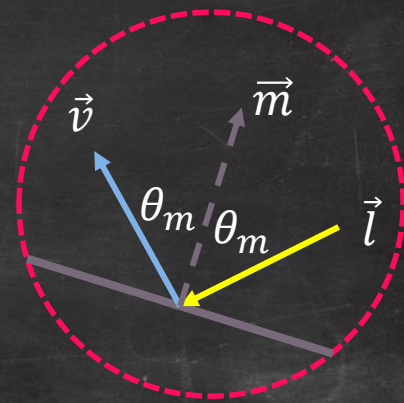
$$\vec{h} = \frac{\vec{l} + \vec{v}}{\|\vec{l} + \vec{v}\|}$$



# Microfacet Model



*microfacet: ideal mirror*



macrogeometry

$$\vec{h} = \frac{\vec{l} + \vec{v}}{\|\vec{l} + \vec{v}\|}$$

# General Microfacet BRDF

*Normal Distribution Function (NDF)*

*Fresnel reflectance*

*Geometric Term*

$$f_r(\vec{l}, \vec{v}) = \text{diffuse} + \frac{D(\theta_h) F(\theta_d) G(\theta_l, \theta_v)}{4 \cos \theta_l \cos \theta_v}$$

The ratio of micro-surface area visible to the **light**, **viewer**

$\theta_l, \theta_v$ : angle between  $\vec{l}, \vec{v}$  and normal  
 $\theta_h$ : angle between normal and  $\vec{h}$   
 $\theta_d$ : difference between  $\vec{l}$  (or  $\vec{v}$ ) and  $\vec{h}$

# Fresnel

- Schlick's approximation

$$F_{Schlick} = F_0 + (1 - F_0)(1 - \overline{\cos \theta_i})^5$$

– Where  $F_0 = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right)^2$

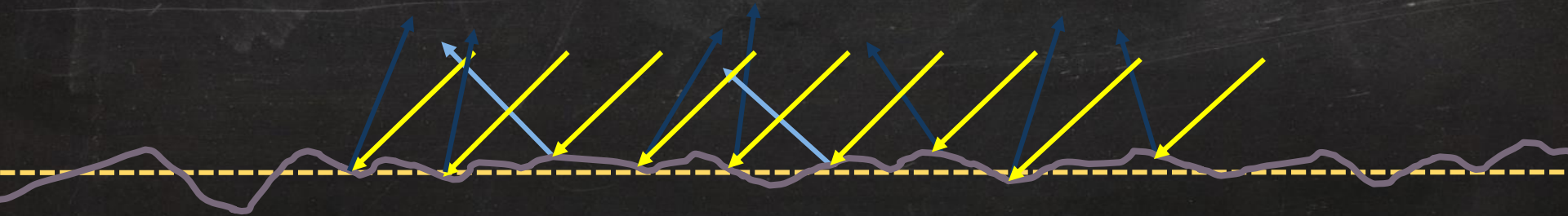
- a.k.a. reflectance at normal, normal reflectance, etc.

? What if  $\eta_2 = \eta_1$

- $F$  should be zero but  $F_{Schlick} = (1 - \overline{\cos \theta_i})^5 \neq 0$

# NDF (Normal Distribution Function)

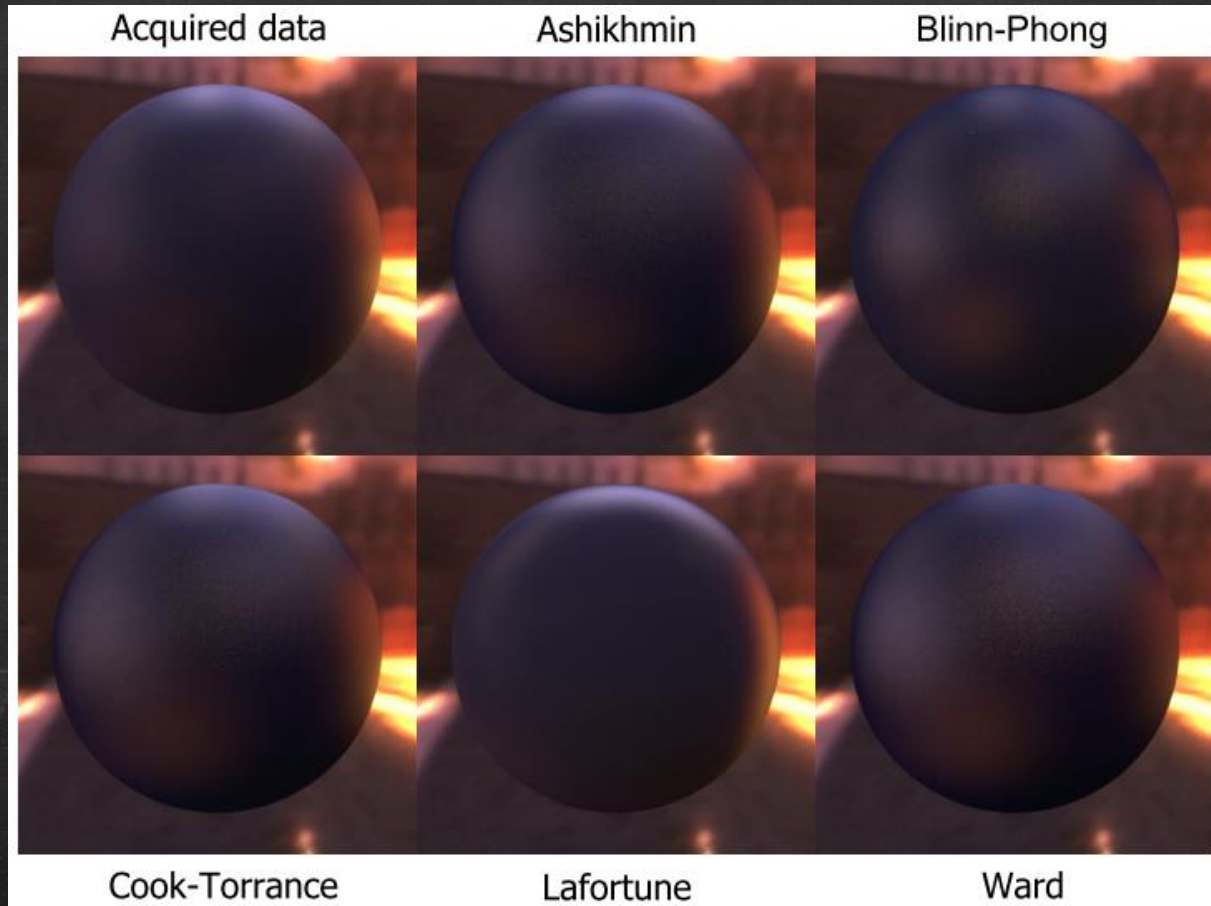
- Half vector  $\vec{h} = \frac{\vec{l} + \vec{v}}{\|\vec{l} + \vec{v}\|}$
- As for perfect mirror microfacets, we can only see those facets whose normal vector  $\vec{m} = \vec{h}$



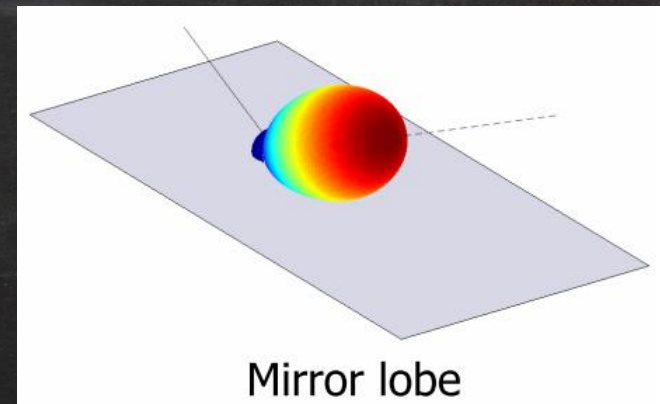
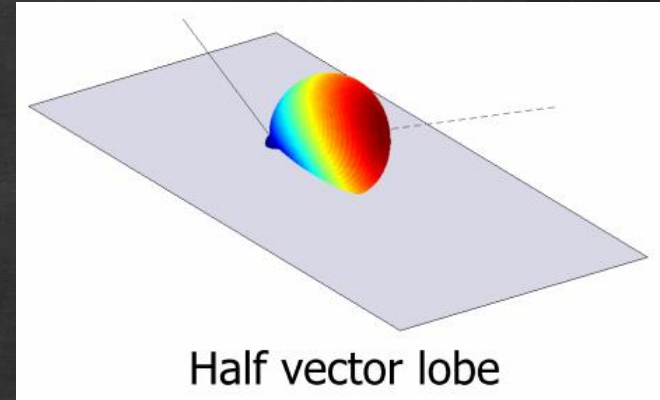
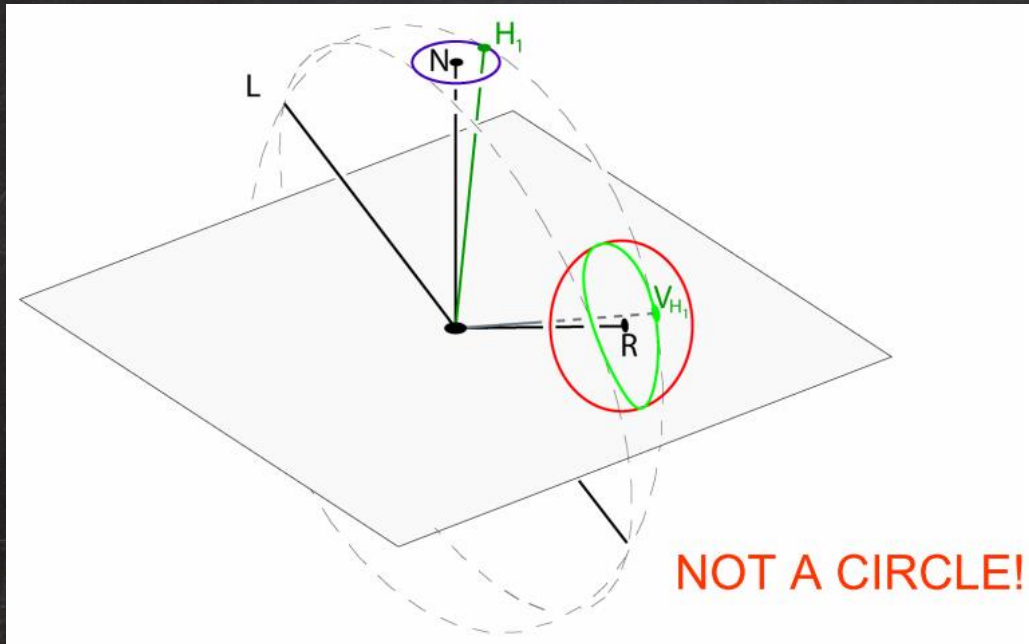
# Highlights at Grazing Angles



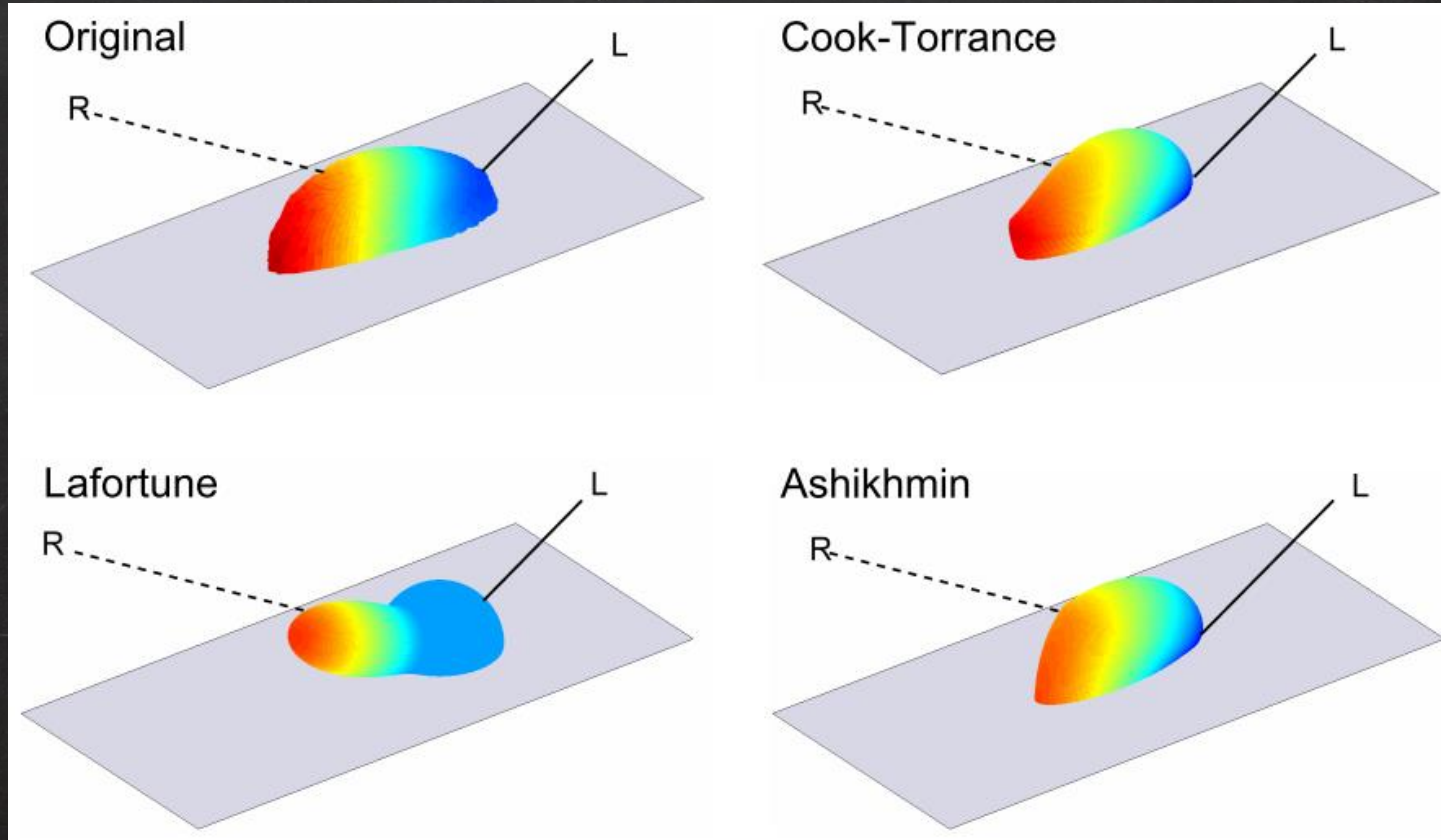
# Data Fitting of Acquired Data



# Highlights at Grazing Angles



# Data Fitting of Acquired Data (Cont'd)





# NDF (Cont'd)

- Measures **area density** of microsurface with respect to microsurface **normal**

$$D(\omega) = \int_{\mathcal{M}} \delta_{\omega}(\omega_m(p_m)) dp_m$$



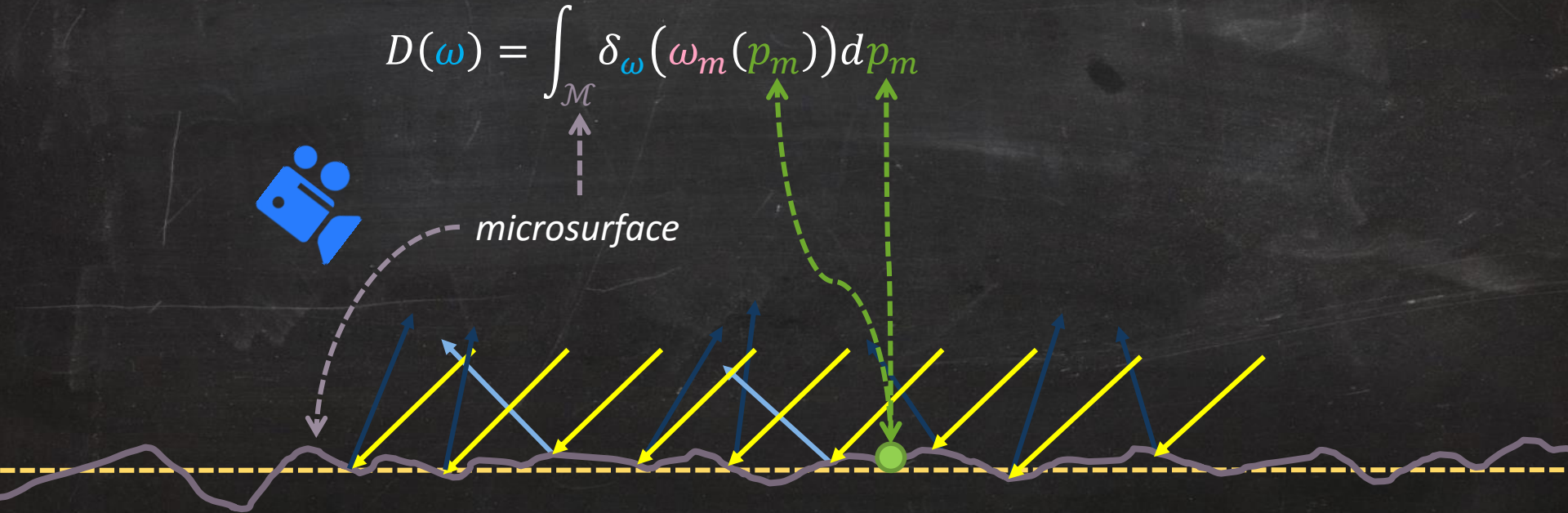
# NDF (Cont'd)

- Measures **area density** of microsurface with respect to microsurface **normal**

$$D(\omega) = \int_{\mathcal{M}} \delta_{\omega}(\omega_m(p_m)) dp_m$$



microsurface



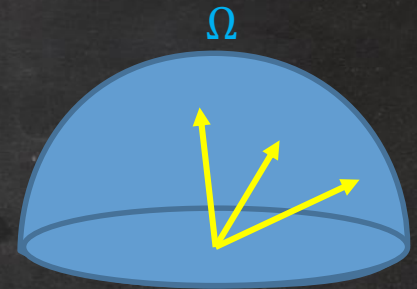
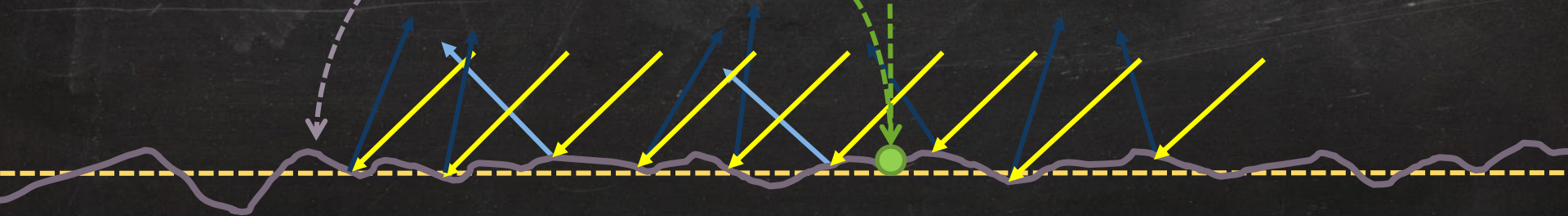
# NDF (Cont'd)

- Measures **area density** of microsurface with respect to microsurface **normal**

$$D(\omega) = \int_{\mathcal{M}} \delta_{\omega}(\omega_m(p_m)) dp_m$$

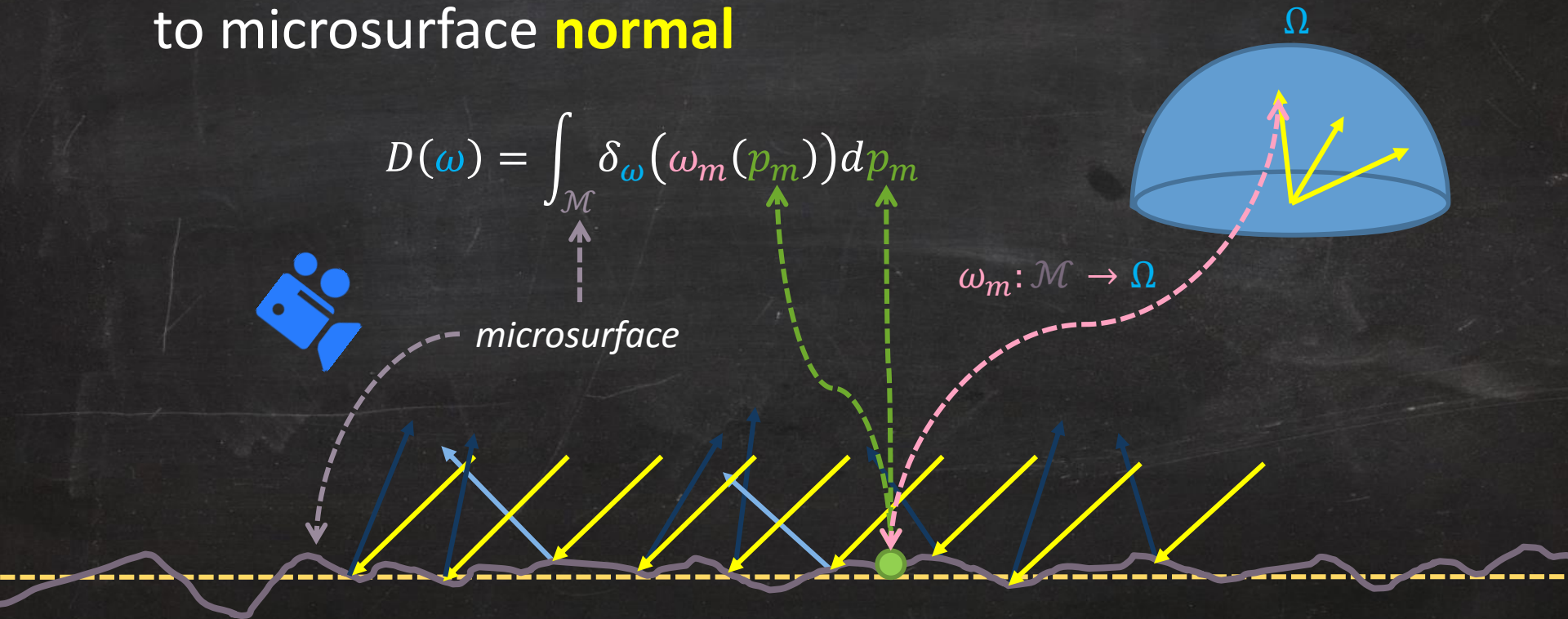


microsurface



# NDF (Cont'd)

- Measures **area density** of microsurface with respect to microsurface **normal**



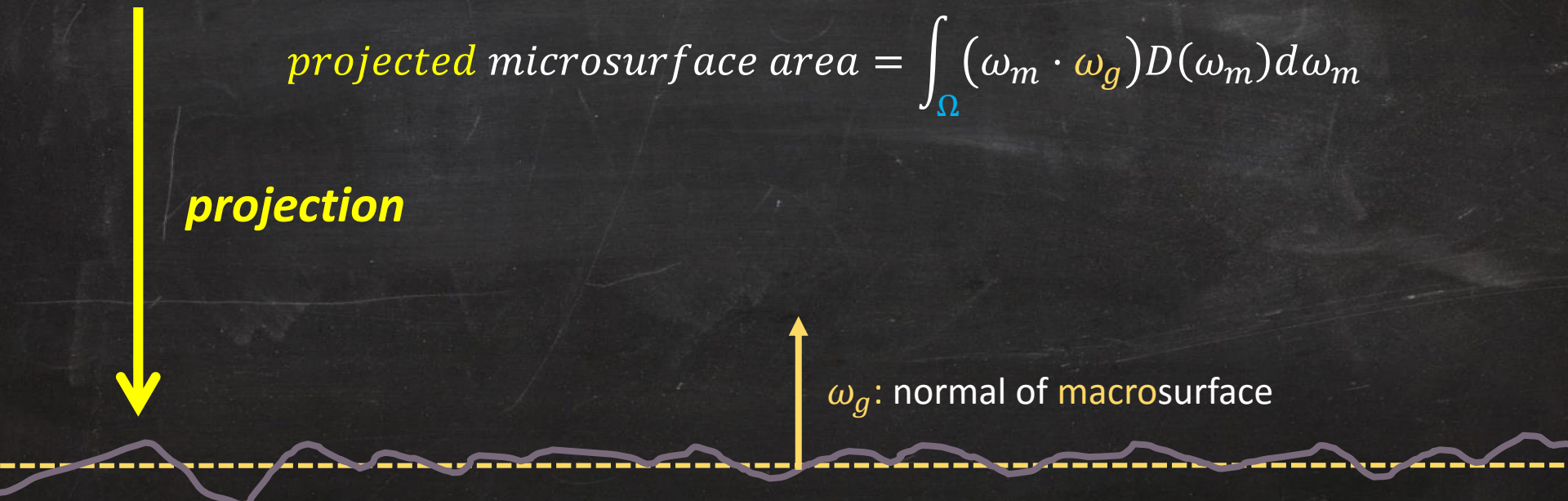
# NDF (Cont'd)

$$\text{microsurface area} = \int_{\mathcal{M}} dp_m = \int_{\Omega} D(\omega_m) d\omega_m$$

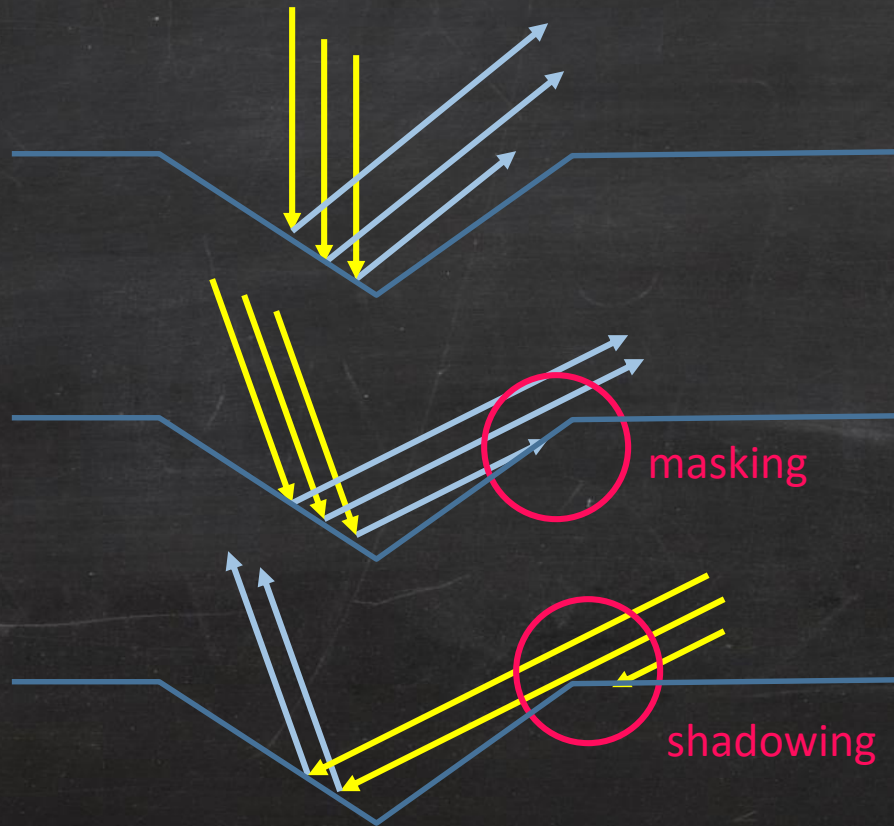
$$\text{projected microsurface area} = \int_{\Omega} (\omega_m \cdot \omega_g) D(\omega_m) d\omega_m$$

**projection**

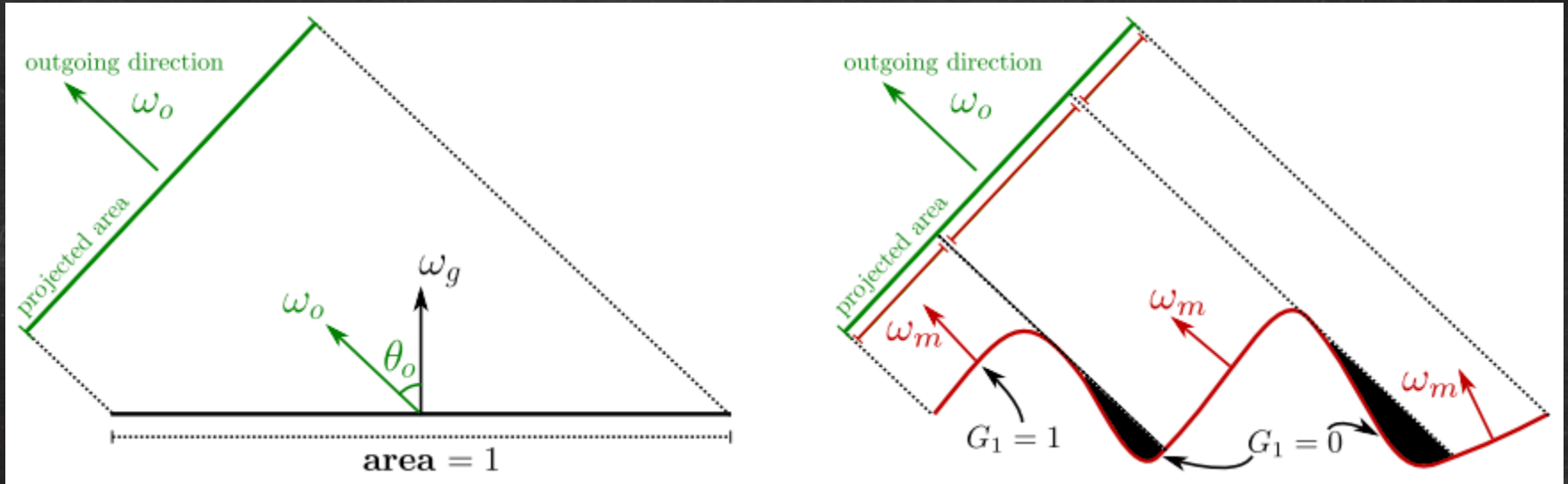
$\omega_g$ : normal of macrosurface



# Masking/Shadowing



# Conservation of Projected Area

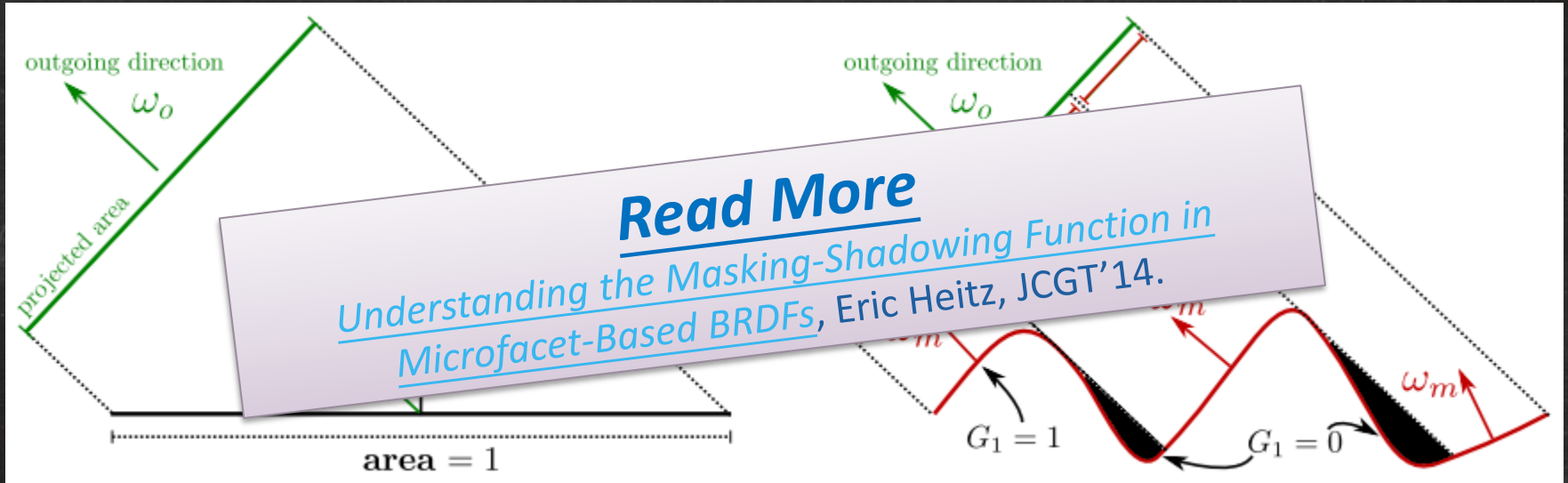


[Heitz '14]

$$\cos \theta_0 = \int_{\Omega} G_1(\omega_0, \omega_m) \langle \omega_0, \omega_m \rangle D(\omega_m) d\omega_m$$

masking function

# Conservation of Projected Area



[Heitz '14]

$$\cos \theta_o = \int_{\Omega} G_1(\omega_o, \omega_m) \langle \omega_o, \omega_m \rangle D(\omega_m) d\omega_m$$

masking function



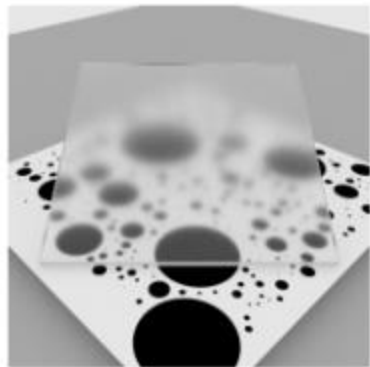
# BRDF Validation

- What makes it physically-based?
  1. Reciprocity:  $f(l, v) = f(v, l)$
  2. Positivity:  $f(l, v) > 0$
  3. Energy conservation:  $\int_{\Omega} f(l, v) \cos \theta_i d\omega_i \leq 1$

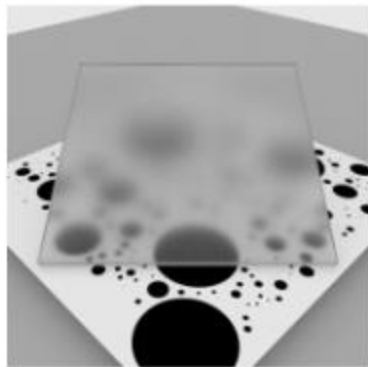
*What do we miss?*

# Multiple Surface Bounces?

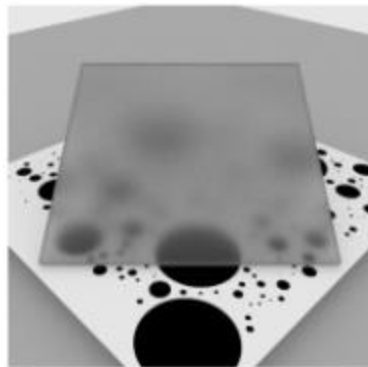
$\alpha = 0.1$



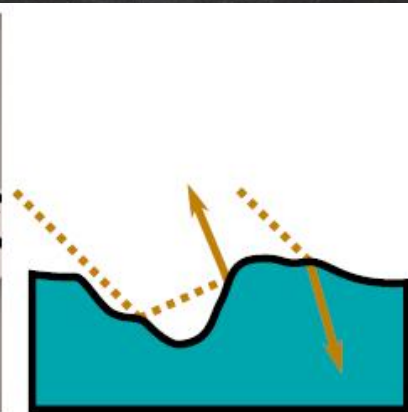
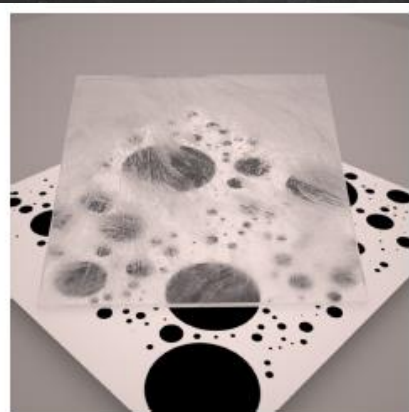
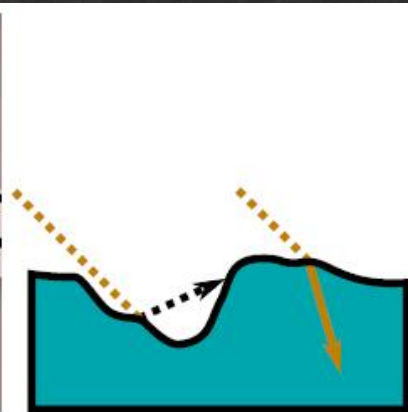
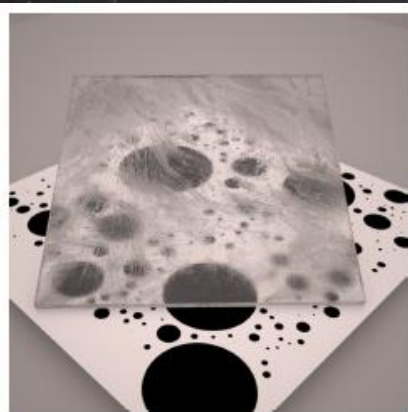
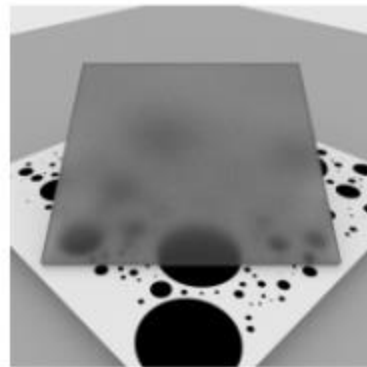
$\alpha = 0.3$



$\alpha = 0.5$



$\alpha = 0.7$



# References

- [Physically-based Rendering](#). SIGGRAPH Course Notes 2011~15.
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